

GRE Math Subject Prep Course: Calculus I

June 7, 2021

1. (Chapter 2 Prob 28)¹ Integrate $\int \frac{x^2}{\sqrt{1-x^2}} dx$.
- (A) $\frac{1}{2}(\arcsin x - \sqrt{1-x^2})$ (B) $\frac{1}{2}(\arcsin x + x\sqrt{1-x^2})$ (C) $\frac{1}{2}(x \arcsin x - \sqrt{1-x^2})$
(D) $\frac{1}{2}(\arcsin x - x\sqrt{1-x^2})$ (E) $\frac{1}{2}(x \arcsin x + \sqrt{1-x^2})$
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2. (Exam III Prob 13)² The value of $I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$ is
- (A) 1 (B) $\frac{\pi}{2}$ (C) 0
(D) $\frac{\pi}{4}$ (E) π
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3. (Chapter 2 Prob 34) The region bounded by the graphs of $y = x^2$ and $y = 6 - |x|$ is revolved around the y -axis. What is the volume of the generated solid?
- (A) $\frac{32}{3}\pi$ (B) 9π (C) 8π
(D) $\frac{20}{3}\pi$ (E) $\frac{16}{3}\pi$
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4. (Exam V Prob 50) Suppose $\int_0^\infty f(x) dx$ exists. Which statements are FALSE?
- I. $\lim_{x \rightarrow \infty} f(x) = 0$
II. $\int_0^\infty |f(x)| dx$ exists
III. $\int_0^\infty [f(x)]^2 dx$ exists
- (A) I only (B) II only (C) I and II only
(D) II and III only (E) I, II and III

¹The problems with “Chapter *” are taken from “Cracking the GRE Mathematics Test”, 4th Edition.

²The problems with “Exam I” – “Exam VI” are taken from the REA book “The Best Test Preparation for the GRE Mathematics Test”, 4th edition.

5. (Week 1 Prob 17)³ For continuous functions $f, g : \mathbb{R} \rightarrow (0, \infty)$, define the relation \sim by $f \sim g$ if and only if

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 1.$$

Suppose that $f \sim g$. Which of these does NOT necessarily follow:

- (A) $f^2 \sim g^2$ (B) $\sqrt{f} \sim \sqrt{g}$ (C) $e^f \sim e^g$
 (D) $f + g \sim 2g$ (E) $g \sim f$
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6. (Week 1 Prob 20) Compute the derivative $\frac{d}{dx} \int_0^{x^2} e^{-t^2} dt$

- (A) e^{-x^4} (B) $2e^{-x^4}$ (C) xe^{-x^4}
 (D) $2xe^{-x^4}$ (E) e^{-x^2}
-

7. (Week 1 Prob 22) Calculate $\lim_{n \rightarrow \infty} \sum_{k=n+1}^{2n} \frac{1}{k}$.

- (A) e^2 (B) 0 (C) $\sin 2$
 (D) $\ln 2$ (E) $\cos 2$
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8. (Week 2 Prob 1) What is the length of the curve $(x(t), y(t)) = (\cos(t), \sin(t))$ for $0 \leq t \leq \pi$?

- (A) 1 (B) 2 (C) π
 (D) 2π (E) π^2
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9. (Week 2 Prob 3) For $n \in \mathbb{N}$, evaluate $\int_0^\infty x^n e^{-x} dx$

- (A) $\ln(n)$ (B) e^{n-1} (C) e^n
 (D) $(n-1)!$ (E) $n!$
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10. (Week 2 Prob 5) Compute $\int \frac{x+2}{x^3 - x^2 + 2x - 2}$.

- (A) $\ln(x-1)$ (B) $\ln(x^2+2)$ (C) $\ln(x-1) - \ln(x^2+2)$
 (D) $\ln(x-1) - 2\ln(x^2+2)$ (E) $\ln(x-1) - \frac{1}{2}\ln(x^2+2)$
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³The problems with “Week *” are taken from the GRE preparation course taught by Christian Parkinson.

11. (Week 2 Prob 8) Compute $\int_0^{\pi/2} \frac{dx}{1 + \tan(x)^{2021}}$.

(A) 2021

(B) $\tan(2021)$

(C) 2021π

(D) 4π

(E) $\frac{\pi}{4}$

12. (Week 2 Prob 10) Let $f(x) = \frac{1}{x}$, for $x \in [1, \infty)$. Find the area and volume of the shape which results from rotating the graph of f about the x -axis

(A) $A = \infty, V = \pi$

(B) $A = \pi, V = \infty$

(C) $A = \pi, V = \pi$

(D) $A = \infty, V = \infty$

(E) $A = \pi, V = 4\pi$

Answer: DDAE CDDC EEEA