

GRE Math Subject Prep Course: Calculus I

June 9, 2021

1. (Exam II Prob 1) Find the sum of the series $\sum_{n=1}^{\infty} \frac{n}{4^{n+1}}$.

(A) $1/12$ (B) $1/9$ (C) $1/6$
(D) $4/3$ (E) $1/3$

2. (Exam I Prob 7) Assuming convergence, find $x = \sqrt{3 + \sqrt{3 + \sqrt{3 + \cdots}}}$.

(A) $\frac{1}{2}(\sqrt{5} + 1)$ (B) $\frac{1}{2}(\sqrt{13} - 1)$ (C) $\frac{1}{2}(\sqrt{5} - 1)$
(D) $\frac{1}{2}(\sqrt{13} + 1)$ (E) $\frac{1}{2}(\sqrt{13} - \sqrt{5})$

3. (Exam I Prob 66) Given that $\sum_{n=1}^{\infty} a_n$ converges to L , which conclusion is valid for $\sum_{n=1}^{\infty} a_n^2$?

(A) It may diverge
(B) It converges absolutely
(C) It converges to $M < L$
(D) It converges to $M > L$
(E) It converges to L^2

4. (Chapter 2 Prob 46) The smallest positive integer x for which the power series $\sum_{n=1}^{\infty} \frac{n!(2n)!}{(3n)!} x^n$ does *not* converge is

(A) 4 (B) 6 (C) 7
(D) 8 (E) 9

5. (Week 2 Prob 11) Evaluate $\lim_{n \rightarrow \infty} (3^n + 5^n)^{\frac{1}{n}}$.

(A) 1 (B) 2 (C) 3
(D) 4 (E) 5

6. (Week 2 Prob 13) Decide whether the following series converge or diverge:

I. $\sum_{n=2}^{\infty} \frac{1}{\ln(n!)}$

II. $\sum_{n=3}^{\infty} \frac{1}{\ln(n)^{\ln(n)}}$

- (A) I and II both converge (B) I converges and II diverges
(C) I diverges and II converges (D) I and II both diverge
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7. (Week 2 Prob 14) Decide whether the following series converge or diverge:

I. $\sum_{n=1}^{\infty} \frac{n!}{2^{n^2}}$

II. $\sum_{n=1}^{\infty} \frac{n^{\sqrt{n}}}{2^n}$

- (A) I and II both converge (B) I converges and II diverges
(C) I diverges and II converges (D) I and II both diverge
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8. (Week 2 Prob 15) Fix an integer $m > 0$. Evaluate the infinite sum

$$\sum_{n=1}^{\infty} \frac{m}{n(n+m)}.$$

(A) $\sum_{k=1}^m k$

(B) $\sum_{k=1}^m k^2$

(C) $\sum_{k=1}^m \frac{1}{k}$

(D) $\sum_{k=1}^m \frac{1}{k^2}$

(E) $\sum_{k=1}^m k$

Answer: BDAC ECAC