GRE Math Subject Prep Course: Calculus III

June 16, 2021

1. (Chapter 3 Prob 4)[1] Given the vector identity $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$, which of the following $\mathbf{V}$ satisfies the equation $\mathbf{A} \times \mathbf{V} = \mathbf{B}$, where $\mathbf{A}$ is a unit vector and $\mathbf{B}$ is a vector orthogonal to $\mathbf{A}$?

   (A) $\mathbf{B} + (\mathbf{A} \times \mathbf{B})$ (B) $\mathbf{B} - (\mathbf{A} \times \mathbf{B})$ (C) $\mathbf{A} \times \mathbf{B}$

   (D) $\mathbf{A} + (\mathbf{A} \times \mathbf{B})$ (E) $\mathbf{A} - (\mathbf{A} \times \mathbf{B})$

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2. (Chapter 3 Prob 11) Consider the following three functions, each of which is defined for all (x, y) in the plane. Which of these functions are continuous at the origin?

$$f_1(x,y) = \begin{cases} \frac{x-y}{x+y} & \text{if } x+y \neq 0 \\ 1 & \text{if } x+y = 0 \end{cases}, \qquad f_2(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases},$$
$$f_3(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

   (A) None (B) f_1 only (C) f_2 only

   (D) f_3 only (E) All three

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3. (Chapter 3 Prob 9) If the curve $z = f(x)$ in the xz-plane is revolved around the x-axis, which of the following is an equation that describes the resulting surface?

   (A) $y^2 + z^2 = |f(x)|$ (B) $z^2 = f(x^2 + y^2)$ (C) $y^2 = f(x^2 + z^2)$

   (D) $x^2 + z^2 = [f(x)]^2$ (E) $y^2 + z^2 = [f(x)]^2$

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4. (Exam II Prob 31)[2] The surface given by $z = x^2 - y^2$ is cut by the plane given by $y = 3x$, producing a curve in the plane. Find the slope of this curve at the point $(1, 3, -8)$.

   (A) 3 (B) -16 (C) $-8\sqrt{\frac{2}{5}}$

   (D) 0 (E) $\frac{18}{\sqrt{10}}$

[1] The problems with "Chapter *" are taken from "Cracking the GRE Mathematics Test", 4th Edition.
[2] The problems with "Exam I" – "Exam VI" are taken from the REA book "The Best Test Preparation for the GRE Mathematics Test", 4th edition.

5. (Exam IV Prob 49) Given $x^2z - 2yz^2 + xy = 0$, find $\dfrac{\partial x}{\partial z}$ at $(1, 1, 1)$.

(A) 0 (B) $\dfrac{4}{3}$ (C) -1

(D) 1 (E) None of these

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6. (Exam II Prob 56) At the point $(2, -1, 2)$ on the surface $z = xy^2$, find a direction vector for the greatest rate of the decrease of z.

(A) $\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ (B) $\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$ (C) $\dfrac{\hat{\mathbf{i}} - 4\hat{\mathbf{j}}}{\sqrt{17}}$

(D) $-\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ (E) $\hat{\mathbf{i}} + \hat{\mathbf{j}}$

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7. (Chapter 3 Prob 21) Which of the following vectors is normal to the surface

$$\log(x + y^2 - z^3) = x - 1$$

at the point where $y = 8$ and $z = 4$?

(A) $\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\,\hat{\mathbf{k}}$ (B) $2\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} + \hat{\mathbf{k}}$ (C) $\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}}$

(D) $-2\,\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\,\hat{\mathbf{k}}$ (E) $\hat{\mathbf{j}} - 3\,\hat{\mathbf{k}}$

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8. (Chapter 3 Prob 23) Find the minimum distance from the origin to the curve

$$3x^2 + 4xy + 3y^2 = 20.$$

(A) 1 (B) $3\sqrt{2}$ (C) 2

(D) $23\sqrt{2}$ (E) $53\sqrt{2}$

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9. (Week 3 Prob 14) Find the equation of the plane containing the origin and the points $(2, 0, 0)$ and $(0, 0, 1)$.

(A) $x = 0$ (B) $y = 0$ (C) $z = 0$

(D) $x + y + z = 0$ (E) $x + y + z = 2$

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10. (Week 3 Prob 15) Let l be the line of intersection for the planes $x+y+z = 3$ and $x-y+z = 5$. Find the equation for the plane containing $(0, 0, 0)$ and perpendicular to l.

(A) $x = y$ (B) $y = z$ (C) $x = z$

(D) $x + y = 0$ (E) $y + z = 0$

Answer: EDECD DECBC