GRE Math Subject Prep Course: Calculus III

June 21, 2021

1. (Exam III Prob 47) Find the Jacobian of the transformation from the xy-plane to the uv-plane defined by

$$\begin{cases} u &= xe^{xy} \\ v &= ye^{xy} \end{cases}$$

(A) $2xye^{xy}$ (B) $(1-x^2y^2)e^{2xy}$ (C) $2e^{2xy}$

(D) $(2xy+1)e^{2xy}$ (E) 0

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2. (Exam II Prob 14) Let $f(x,y) = x^3 - axy + y^2 - x$. Find the greatest lower bound for a so that $f(x,y)$ has a relative minimum point.

(A) 0 (B) $\sqrt{48}$ (C) 12

(D) 6 (E) Does not exist

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3. (Chapter 3 Prob 27) Find $\iint_R z\,dx\,dy$, where $z = 8xy$, and R is the region in the first quadrant of $\mathbb{R}^2$ bounded by the two axes and the unit circle.

(A) $\frac{1}{2}$ (B) 1 (C) 2

(D) 4 (E) 8

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4. (Exam V Prob 17) The length of the curve $x(t) = e^t \cos t, y(t) = -e^t \sin t$ for $0 \le t \le 1$ is

(A) $2(e-1)$ (B) $\sqrt{2}(e-1)$ (C) e

(D) $2e$ (E) $\sqrt{2}$

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5. (Chapter 3 Prob 25) If $\mathbf{F} = (2y-2x)\,\hat{\mathbf{i}} + (x^2+y)\,\hat{\mathbf{j}}$, find the value of $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is the portion of the parabola $y = x^2$, directed from $(-1,1)$ to the origin.

(A) -1 (B) 0 (C) 1

(D) 2 (E) 3

6. (Chapter 3 Prob 26) Let C be the portion of the astroid $x^{2/3} + y^{2/3} = 1$ from $(1, 0)$ to $(0, 1)$, which can be parameterized by the equations

$$x = \cos^3 t, y = \cos^3 t$$

as t increases from 0 to $\frac{\pi}{2}$. Evaluate the integral:

$$\int_C (y \cos xy - 1)dx + (1 + x \cos xy)dy$$

(A) -2 (B) -1 (C) 1

(D) $\frac{1}{2}\pi - 1$ (E) 2

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7. (Week 3 Prob 16) Find all functions $f(x, y)$ satisfying $\frac{\partial f}{\partial x}(x, y) = 2x + y$, $\frac{\partial f}{\partial y}(x, y) = x + 2y$.

(A) $x^2 + xy + y^2 + C$ (B) $x^2 + 2xy + y^2 + C$ (C) $2xy + y^2 + C$

(D) $x^3 + xy + y + C$ (E) $x^3 + y^3 + C$

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8. (Week 3 Prob 17) Find the point on the plane $2x + y + 3z = 3$ which is closest to the origin.

(A) $(\frac{3}{14}, \frac{3}{14}, \frac{9}{14})$ (B) $(\frac{3}{7}, \frac{3}{14}, \frac{9}{14})$ (C) $(0, 0, 0)$

(D) $(\frac{3}{7}, \frac{3}{7}, \frac{9}{14})$ (E) $(1, 1, 1)$

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9. (Week 3 Prob 19) Set up an integral which represents the volume of the solid bounded above by the graph of $z = 6 - x^2 - 2y^2$ and below by the graph of $z = -2 + x^2 + 2y^2$.

(A) $\displaystyle\int_0^1 \int_{-\sqrt{(4-x^2)/2}}^{\sqrt{(4-x^2)/2}} (6 - x^2 - 2y^2) dx\, dy$

(B) $\displaystyle\int_0^{\sqrt{2}} \int_{-\sqrt{(4-x)^2/2}}^{\sqrt{(4-x)^2/2}} (-2 + x^2 + 2y^2) dx\, dy$

(C) $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{(4-x^2)/2}}^{\sqrt{(4-x^2)/2}} (8 - 2x^2 - 4y^2) dx\, dy$

(D) $\displaystyle\int_0^{\sqrt{2}} \int_{-\sqrt{(4-x)^2/2}}^{\sqrt{(4-x)^2/2}} (8 - 2x^2 - 4y^2) dx\, dy$

(E) $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{(4-x^2)/2}}^{\sqrt{(4-x^2)/2}} (6 - x^2 - 2y^2) dx\, dy$

10. (Week 3 Prob 20) Minimize the function $f(x, y, z) = x + 4z$ on the curve $x^2 + y^2 + z^2$.

(A) $-\sqrt{\frac{1}{17}}$ (B) $-17\sqrt{\frac{1}{17}}$ (C) $-\sqrt{\frac{2}{17}}$

(D) $-17\sqrt{\frac{2}{17}}$ (E) $-17\sqrt{\frac{3}{17}}$

Answer: DEBBC EABCD