GRE Math Subject Prep Course: Calculus III

June 23, 2021

1. (Exam V Prob 17) The length of the curve $x(t) = e^t \cos t$, $y(t) = -e^t \sin t$ for $0 \le t \le 1$ is

- (A) 2(e-1) (B) $\sqrt{2}(e-1)$ (C) e
- (D) 2e (E) $\sqrt{2}$
- 2. (Chapter 3 Prob 25) If $\mathbf{F} = (3y 2x) \hat{\mathbf{i}} + (x^2 + y) \hat{\mathbf{j}}$, find the value of $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the portion of the parabola $y = x^2$, directed from (-1, 1) to the origin.

(A) -1	(B) 0	(C)	1
(D) 2	(E) 3		

3. (Chapter 3 Prob 26) Let C be the portion of the astroid $x^{2/3} + y^{2/3} = 1$ from (1,0) to (0,1), which can be parameterized by the equations

$$x = \cos^3 t, y = \cos^3 t$$

as t increases from 0 to $\frac{\pi}{2}$. Evaluate the integral:

$$\int_{C} (y \cos xy - 1) dx + (1 + x \cos xy) dy$$
(A) -2
(B) -1
(C) 1
(D) $\frac{1}{2}\pi - 1$
(E) 2

4. (Week 3 Prob 22) Find the integral of $f(x, y) = e^{y^2}$ over the triangular region bounded by the graph of y = |x| for $x \in [-2, 2]$ and the link y = 2.

(A) $e - 1$	(B) $e^2 - e$	(C) $e^4 - e^2$
(D) $e^4 - 1$	(E) $e^2 - 1$	

5. (Week 3 Prob 23) Let $D = \{(x, y) \in \mathbb{R} : x \ge 0, y \ge 0\}$. Calculate

$$\iint_D e^{-(x^2+y^2)} \, dx \, dy$$

(A) $\frac{\pi}{2}$	(B) $\frac{\pi}{4}$	(C)
(D) $\frac{\pi^2}{2}$	(E) $\frac{\pi^2}{4}$	

6. (Week 3 Prob 26) Let \mathcal{C} be the ellipse given by $(\frac{x}{a})^2 + (\frac{y}{b})^2 = 1$ (a, b > 0). Calculate

$$\oint_{\mathcal{C}} (-y) dx + x dy.$$

 π

(A) ab	(B) πab	(C) $2\pi ab$
(D) $\pi^2 ab$	(E) $2\pi^2 ab$	

7. (Week 3 Prob 27) Let C be the triangle with vertices (0,0), (1,0), (1,2). Find the path integral of $\mathbf{F}(x,y) = (xy, x^2y^3)$ around this curve.

(A) $\frac{2}{3}$	(B) $\frac{1}{3}$	(C) 1
(D) $\frac{1}{2}$	(E) $\frac{3}{2}$	

8. (Week 3 Prob 28) What is the flux of $\mathbf{F}(x, y, z) = (x, y, z)$ through the surface $z = \sqrt{1 - x^2 - y^2}$ with normal pointing upward?

(A) π^2	(B) $2\pi^2$	(C) 3

(D) π (E) 2π

Answer: BCED BCAE