# GRE Math Subject Prep Course: Calculus III 

June 23, 2021

1. (Exam V Prob 17) The length of the curve $x(t)=e^{t} \cos t, y(t)=-e^{t} \sin t$ for $0 \leq t \leq 1$ is
(A) $2(e-1)$
(B) $\sqrt{2}(e-1)$
(C) $e$
(D) $2 e$
(E) $\sqrt{2}$
2. (Chapter 3 Prob 25) If $\mathbf{F}=(3 y-2 x) \hat{\mathbf{i}}+\left(x^{2}+y\right) \hat{\mathbf{j}}$, find the value of $\int_{C} \mathbf{F} \cdot d \mathbf{r}$, where $C$ is the portion of the parabola $y=x^{2}$, directed from $(-1,1)$ to the origin.
(A) -1
(B) 0
(C) 1
(D) 2
(E) 3
3. (Chapter 3 Prob 26) Let $C$ be the portion of the astroid $x^{2 / 3}+y^{2 / 3}=1$ from $(1,0)$ to $(0,1)$, which can be parameterized by the equations

$$
x=\cos ^{3} t, y=\cos ^{3} t
$$

as $t$ increases from 0 to $\frac{\pi}{2}$. Evaluate the integral:

$$
\int_{C}(y \cos x y-1) d x+(1+x \cos x y) d y
$$

(A) -2
(B) -1
(C) 1
(D) $\frac{1}{2} \pi-1$
(E) 2
4. (Week 3 Prob 22) Find the integral of $f(x, y)=e^{y^{2}}$ over the triangular region bounded by the graph of $y=|x|$ for $x \in[-2,2]$ and the link $y=2$.
(A) $e-1$
(B) $e^{2}-e$
(C) $e^{4}-e^{2}$
(D) $e^{4}-1$
(E) $e^{2}-1$
5. (Week 3 Prob 23) Let $D=\{(x, y) \in \mathbb{R}: x \geq 0, y \geq 0\}$. Calculate

$$
\iint_{D} e^{-\left(x^{2}+y^{2}\right)} d x d y
$$

(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{4}$
(C) $\pi$
(D) $\frac{\pi^{2}}{2}$
(E) $\frac{\pi^{2}}{4}$
6. (Week 3 Prob 26) Let $\mathcal{C}$ be the ellipse given by $\left(\frac{x}{a}\right)^{2}+\left(\frac{y}{b}\right)^{2}=1(a, b>0)$. Calculate

$$
\oint_{\mathcal{C}}(-y) d x+x d y
$$

(A) $a b$
(B) $\pi a b$
(C) $2 \pi a b$
(D) $\pi^{2} a b$
(E) $2 \pi^{2} a b$
7. (Week 3 Prob 27) Let $\mathcal{C}$ be the triangle with vertices $(0,0),(1,0),(1,2)$. Find the path integral of $\mathbf{F}(x, y)=\left(x y, x^{2} y^{3}\right)$ around this curve.
(A) $\frac{2}{3}$
(B) $\frac{1}{3}$
(C) 1
(D) $\frac{1}{2}$
(E) $\frac{3}{2}$
8. (Week 3 Prob 28) What is the flux of $\mathbf{F}(x, y, z)=(x, y, z)$ through the surface $z=\sqrt{1-x^{2}-y^{2}}$ with normal pointing upward?
(A) $\pi^{2}$
(B) $2 \pi^{2}$
(C) 3
(D) $\pi$
(E) $2 \pi$

Answer: BCED BCAE

