# GRE Math Subject Prep Course: Differential Equations 

July 7, 2021

1. (Chapter 4 Prob 1$)^{1}$ Let $y=f(x)$ be the solution of the equation

$$
\frac{d y}{d x}=\frac{x^{2}}{x^{2}+1}
$$

such that $y=0$ when $x=0$. What is the value of $f(1)$ ?
(A) $1-\log 2$
(B) $1+\log 2$
(C) 1
(D) $\log 2$
(E) $\frac{1}{4}(4-\pi)$
2. (Week 4 Prob 7) $)^{2}$ Find all solutions of the equation $y y^{\prime \prime}-2\left(y^{\prime}\right)^{2}=0$ which pass through $x=1, y=1$.
(A) $y=\frac{1}{x}$
(B) $y=\frac{1}{1+C(1-x)}$ for $C>0$
(C) $y=C(x-1)(x-2)+1$ for $C>0$
(D) $y=C(\log x)^{2}+1$ for $C>0$
(E) $y=\frac{C x^{2}}{x+C-1}$ for $C>0$
3. (Chapter 4 Prob 5) If $a$ is a positive constant, let $y=f(x)$ be the solution of the equation

$$
y^{\prime \prime \prime}-a y^{\prime \prime}+a^{2} y^{\prime}-a^{3} y=0
$$

such that $f(0)=1, f^{\prime}(0)=0$, and $f^{\prime \prime}(0)=a^{2}$. How many positive values of $x$ satisfies the equation $f(x)=0$ ?
(A) 0
(B) 1
(C) 2
(D) 3
(E) more than 3
4. (Week 4 Prob 3) A tank initially contains a salt solution of 3 grams of salt dissolved in 100 liters of water. A salt solution containing 0.02 grams of salt per liter is pumped into the tank at 4 liters per minute. The tank is also draining at 4 liters per minute. Assuming the mixing is instantaneous, how many grams of salt are in the tank after 100 minutes?
(A) $1+e^{-3}$
(B) $1+e^{-4}$
(C) $2+e^{-3}$
(D) $2+e^{-4}$
(E) $3+e^{-3}$

[^0]5. (Week 4 Prob 10) Which of the following are linear subspaces of the continuous functions from $\mathbb{R}$ to $\mathbb{R}$ ?
I. $\left\{f: f\right.$ is twice differentiable and $f^{\prime \prime}(x)-2 f^{\prime}(x)+3 f(x)=0$ for all $\left.x\right\}$
II. $\left\{g: g\right.$ is twice differentiable and $g^{\prime \prime}(x)=3 g^{\prime}(x)$ for all $\left.x\right\}$
III. $\left\{h: h\right.$ is twice differentiable and $h^{\prime \prime}(x)=h(x)+1$ for all $\left.x\right\}$
(A) I only
(B) I and II only
(C) I, II and III
(D) II and III only
(E) None of above
6. (Practice Prob 44) ${ }^{3}$ If $y$ is a real-valued function defined on the real line and satisfying the initial value problem
\[

$$
\begin{aligned}
y^{\prime}+x y & =x \\
y(0) & =-1 .
\end{aligned}
$$
\]

Then $\lim _{x \rightarrow-\infty} y(x)=$
(A) 0
(B) 1
(C) -1
(D) $\infty$
(E) $-\infty$
7. (Week 4 Prob 4) Find the solution of $x d y+\left(y-x e^{x}\right) d x=0$ which passes through the point $(1,0)$.
(A) $y=x^{2}-1$
(B) $y=(x-1)(x-2)$
(C) $y=e^{x}-e^{x} / x$
(D) $y=\log x$
(E) $y=x e^{x}-e^{x}$
8. (Chapter 4 Prob 7) Let $y=f(x)$ be the solution of the equation

$$
\frac{d y}{d x}+\frac{y}{x}=\sin x
$$

such that $f(\pi)=1$. What is the value of $f\left(\frac{1}{2} \pi\right)$ ?
(A) $\frac{2}{\pi}-1$
(B) $\frac{2}{\pi}$
(C) $\frac{2}{\pi}+1$
(D) $\frac{\pi}{2}$
(E) $\frac{\pi}{2}+1$

[^1]9. (Chapter 4 Prob 8) Let $y=f(x)$ be the solution of the equation
$$
\frac{d^{4} y}{d x^{4}}=\frac{d^{2} y}{d x^{2}}
$$
such that $f(0)=f^{\prime}(0)=f^{\prime \prime}(0)=0$ and $f^{\prime \prime \prime}(0)=-1$. What is $f(x)$ ?
(A) $x-\cosh x$
(B) $x-\sinh x$
(C) $x+\cosh x$
(D) $x+\sinh x$
(E) $\cosh x+\sinh x$
10. (Chapter 4 Prob 6) Let $g: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable and integrable function. The integral curve of the differential equation
$$
[y+g(x)] d x+[x-g(y)] d y=0
$$
that passes through the point $(1,1)$ must also pass through which of the following points?
(A) $(0,0)$
(B) $\left(2, \frac{1}{2}\right)$
(C) $\left(\frac{1}{2}, 2\right)$
(D) $(-1,-1)$
(E) $(0,1)$
11. (Chapter 4 Prob 2) A population of bacteria grows at a rate proportional to the number present. After two hours, the population has tripled. After two more hours elapse, the population will have increased by a factor of $k$. What is the value of $k$ ?
(A) 6
(B) 8
(C) 9
(D) 27
(E) 81
12. (Chapter 4 Prob 10) Given the following differential equations has an integrating factor of the form $\mu(x, y)=x^{m} y^{n}$, determine its general solution.
$$
\left(3 x y^{2}-5 y\right) d x+\left(2 x^{2} y-3 x\right) d y=0
$$
(A) $x^{4} y^{2}\left(\frac{1}{2} x y-1\right)=c$
(B) $x^{4} y^{2}(x y-1)=c$
(C) $x^{4} y^{2}(2 x y-1)=c$
(D) $x^{5} y^{3}\left(\frac{1}{2} x y-1\right)=c$
(E) $x^{5} y^{3}(2 x y-1)=c$

Answer: EBAD BBCB BDCD


[^0]:    ${ }^{1}$ The problems with "Chapter *" are taken from "Cracking the GRE Mathematics Test", 4th Edition.
    ${ }^{2}$ The problems with "Week *" are taken from Christian Parkinson's GRE problem lists

[^1]:    ${ }^{3}$ The problems with "Practice Book" are taken from the mathematics test practice book by ETS, which can be found at http://www.ets.org/Media/Tests/GRE/pdf/Math.pdf

