GRE Math Subject Prep Course: Differential Equations

July 7, 2021

1. (Chapter 4 Prob 1)¹ Let y = f(x) be the solution of the equation

$$\frac{dy}{dx} = \frac{x^2}{x^2 + 1}$$

such that y = 0 when x = 0. What is the value of f(1)?

- (A) $1 \log 2$ (B) $1 + \log 2$ (C) 1 (D) $\log 2$ (E) $\frac{1}{4}(4 - \pi)$
- 2. (Week 4 Prob 7)² Find all solutions of the equation $yy'' 2(y')^2 = 0$ which pass through x = 1, y = 1.
 - (A) $y = \frac{1}{x}$ (B) $y = \frac{1}{1+C(1-x)}$ for C > 0(C) y = C(x-1)(x-2) + 1 for C > 0(D) $y = C(\log x)^2 + 1$ for C > 0(E) $y = \frac{Cx^2}{x+C-1}$ for C > 0
- 3. (Chapter 4 Prob 5) If a is a positive constant, let y = f(x) be the solution of the equation $y''' - ay'' + a^2y' - a^3y = 0$

such that f(0) = 1, f'(0) = 0, and $f''(0) = a^2$. How many positive values of x satisfies the equation f(x) = 0?

- (A) 0 (B) 1 (C) 2
- (D) 3 (E) more than 3
- 4. (Week 4 Prob 3) A tank initially contains a salt solution of 3 grams of salt dissolved in 100 liters of water. A salt solution containing 0.02 grams of salt per liter is pumped into the tank at 4 liters per minute. The tank is also draining at 4 liters per minute. Assuming the mixing is instantaneous, how many grams of salt are in the tank after 100 minutes?
 - (A) $1 + e^{-3}$ (B) $1 + e^{-4}$ (C) $2 + e^{-3}$
 - (D) $2 + e^{-4}$ (E) $3 + e^{-3}$

 $^{^1\}mathrm{The}$ problems with "Chapter *" are taken from "Cracking the GRE Mathematics Test", 4th Edition.

 $^{^2 \}mathrm{The}$ problems with "Week *" are taken from Christian Parkinson's GRE problem lists

- - I. $\{f : f \text{ is twice differentiable and } f''(x) 2f'(x) + 3f(x) = 0 \text{ for all } x\}$
 - II. $\{g : g \text{ is twice differentiable and } g''(x) = 3g'(x) \text{ for all } x\}$
 - III. $\{h : h \text{ is twice differentiable and } h''(x) = h(x) + 1 \text{ for all } x\}$
 - (A) I only(B) I and II only(C) I, II and III(D) II and III only(E) None of above
- 6. (Practice Prob 44)³ If y is a real-valued function defined on the real line and satisfying the initial value problem

$$y' + xy = x$$
$$y(0) = -1.$$

Then $\lim_{x \to -\infty} y(x) =$
(A) 0 (B) 1 (C) -1
(D) ∞ (E) $-\infty$

- 7. (Week 4 Prob 4) Find the solution of $xdy + (y xe^x)dx = 0$ which passes through the point (1,0).
 - (A) $y = x^2 1$ (B) y = (x 1)(x 2) (C) $y = e^x e^x/x$ (D) $y = \log x$ (E) $y = xe^x - e^x$

8. (Chapter 4 Prob 7) Let y = f(x) be the solution of the equation

$$\frac{dy}{dx} + \frac{y}{x} = \sin x$$

such that $f(\pi) = 1$. What is the value of $f(\frac{1}{2}\pi)$?

- (A) $\frac{2}{\pi} 1$ (B) $\frac{2}{\pi}$ (C) $\frac{2}{\pi} + 1$
- (D) $\frac{\pi}{2}$ (E) $\frac{\pi}{2} + 1$

 $^{^{3}{\}rm The}$ problems with "Practice Book" are taken from the mathematics test practice book by ETS, which can be found at http://www.ets.org/Media/Tests/GRE/pdf/Math.pdf

9. (Chapter 4 Prob 8) Let y = f(x) be the solution of the equation

$$\frac{d^4y}{dx^4} = \frac{d^2y}{dx^2}$$

such that f(0) = f'(0) = f''(0) = 0 and f'''(0) = -1. What is f(x)?

(A) $x - \cosh x$ (B) $x - \sinh x$ (C) $x + \cosh x$ (D) $x + \sinh x$ (E) $\cosh x + \sinh x$

10. (Chapter 4 Prob 6) Let $g : \mathbb{R} \to \mathbb{R}$ be a differentiable and integrable function. The integral curve of the differential equation

$$[y + g(x)]dx + [x - g(y)]dy = 0$$

that passes through the point (1, 1) must also pass through which of the following points?

- (A) (0,0) (B) $(2,\frac{1}{2})$ (C) $(\frac{1}{2},2)$
- (D) (-1, -1) (E) (0, 1)
- 11. (Chapter 4 Prob 2) A population of bacteria grows at a rate proportional to the number present. After two hours, the population has tripled. After two more hours elapse, the population will have increased by a factor of k. What is the value of k?

(A) 6	(B) 8	(C)	9
(D) 27	(E) 81		

12. (Chapter 4 Prob 10) Given the following differential equations has an integrating factor of the form $\mu(x, y) = x^m y^n$, determine its general solution.

$$(3xy^2 - 5y)dx + (2x^2y - 3x)dy = 0$$

(A) $x^4 y^2 (\frac{1}{2}xy - 1) = c$ (B) $x^4 y^2 (xy - 1) = c$ (C) $x^4 y^2 (2xy - 1) = c$

(D)
$$x^5 y^3 (\frac{1}{2}xy - 1) = c$$
 (E) $x^5 y^3 (2xy - 1) = c$

Answer: EBAD BBCB BDCD