GRE Math Subject Prep Course: Linear Algebra

June 28, 2021

1. (Chapter 5 Prob 5)¹ If the matrices

$$\begin{pmatrix} 3 & -2 & -2 \\ 1 & -1 & -1 \\ 3 & -1 & -2 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & a & 0 \\ -1 & b & 1 \\ 2 & c & -1 \end{pmatrix}$$

are inverses of each other, what is the value of c?

(\mathbf{A})	-3	(B) -2	(C)	0
(D)	2	(E) 3		

2. $(\text{Exam II Prob } 18)^2$ In an homogeneous system of 5 linear equations with 7 unknowns, the rank of the coefficient matrix is 4. The maximum number of independent solution vectors is

(A) 5	(B) 2	(C) 4
(D) 1	(E) 3	

- 3. (Exam II Prob 40) If A is a square matrix of order $n \ge 4$, and $a_{ij} = i + j$ represents the entry in row i and column j, then the rank of A is always
 - (C) n-2(A) 1 (B) 2
 - (D) n-1(E) none of these
- 4. (Exam II Prob 15) Given that S and T are subspaces of a vector space, which of the following is also a subspace?
 - (A) $S \cap T$ (B) $S \cup T$ (C) 2S
 - (D) Both (A) and (C) (E) Both (B) and (C)

¹The problems with "Chapter *" are taken from "Cracking the GRE Mathematics Test", 4th Edition. ²The problems with "Exam I" – "Exam VI" are taken from the REA book "The Best Test Preparation for the GRE Mathematics Test", 4th edition.

5. (Exam IV Prob 35) If T is a linear transformation mapping vectors (1, 0, 0), (0, 1, 0) and (0, 0, 1) to the vectors (1, 2, 3), (2, 3, 1) and (1, 1, -2) respectively which vector is the image of the vector (3, -2, 1) under T?

(A) (1,1,7) (B) (1,0,5) (C) (0,1,5)

(D) (0,1,9) (E) (1,7,0)

6. (Chapter 5 Prob 16) Define linear operator S and T on the xy-plane (\mathbb{R}^2) as follows: S rotates each vector 90° counterclockwise, and T reflects each vector through the y-axis. If ST and TS denote the compositions $S \circ T$ and $T \circ S$, respectively, and I is the identity map, which of the following is true?

(A) ST = I (B) ST = -I (C) TS = I(D) ST = TS (E) ST = -TS

- 7. (Chapter 5 Prob 15) Let $T : \mathbb{R}^5 \to \mathbb{R}^3$ be a linear transformation whose kernel is a threedimensional subspace of \mathbb{R}^5 . The set $\{T(x) : x \in \mathbb{R}^5\}$ is
 - (A) the trivial subspace
 - (B) a line through the origin
 - (C) a plane through the origin
 - (D) all of \mathbb{R}^3
 - (E) Cannot be determined from the information given
- 8. (Chapter 5 Prob 3) Let A, B and C be real 2×2 matrices, and let 0 denote the 2×2 zero matrix. Which of the following statements is/are true?

I. $A^2 = 0 \Rightarrow A = 0$ II. $AB = AC \Rightarrow B = C$ III. A is invertible and $A = A^{-1} \Rightarrow A = I$ or A = -I(A) I only (B) I and III only (C) II and III only (D) III only (E) none of the above

- 9. (Week 4 Prob 11) If V, W are 2-dimensional subspaces of \mathbb{R}^4 , what are the possible dimensions of $V \cap W$?
 - (A) 0 (B) 0,1 (C) 0,1,2
 - (D) 1,2 (E) 2

- 10. (Week 4 Prob 12) Suppose that V is the vector space of real 2×3 matrices. If T is a linear transformation from V onto \mathbb{R}^4 , what is the dimension of the null space of T?
 - (A) 0 (B) 1 (C) 2
 - (D) 3 (E) 4

Answer: AEBD CECE CC