# GRE Math Subject Prep Course: Linear Algebra 

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1. (Chapter 5 Prob 5) ${ }^{1}$ If the matrices

$$
\left(\begin{array}{lll}
3 & -2 & -2 \\
1 & -1 & -1 \\
3 & -1 & -2
\end{array}\right) \text { and }\left(\begin{array}{ccc}
1 & a & 0 \\
-1 & b & 1 \\
2 & c & -1
\end{array}\right)
$$

are inverses of each other, what is the value of $c$ ?
(A) -3
(B) -2
(C) 0
(D) 2
(E) 3
2. (Exam II Prob 18) $)^{2}$ In an homogeneous system of 5 linear equations with 7 unknowns, the rank of the coefficient matrix is 4 . The maximum number of independent solution vectors is
(A) 5
(B) 2
(C) 4
(D) 1
(E) 3
3. (Exam II Prob 40) If $A$ is a square matrix of order $n \geq 4$, and $a_{i j}=i+j$ represents the entry in row $i$ and column $j$, then the rank of $A$ is always
(A) 1
(B) 2
(C) $n-2$
(D) $n-1$
(E) none of these
4. (Exam II Prob 15) Given that $S$ and $T$ are subspaces of a vector space, which of the following is also a subspace?
(A) $S \cap T$
(B) $S \cup T$
(C) $2 S$
(D) Both (A) and (C)
(E) Both (B) and (C)

[^0]5. (Exam IV Prob 35) If $T$ is a linear transformation mapping vectors $(1,0,0),(0,1,0)$ and $(0,0,1)$ to the vectors $(1,2,3),(2,3,1)$ and $(1,1,-2)$ respectively which vector is the image of the vector $(3,-2,1)$ under $T$ ?
(A) $(1,1,7)$
(B) $(1,0,5)$
(C) $(0,1,5)$
(D) $(0,1,9)$
(E) $(1,7,0)$
6. (Chapter 5 Prob 16) Define linear operator $S$ and $T$ on the $x y$-plane ( $\mathbb{R}^{2}$ ) as follows: $S$ rotates each vector $90^{\circ}$ counterclockwise, and $T$ reflects each vector through the $y$-axis. If $S T$ and $T S$ denote the compositions $S \circ T$ and $T \circ S$, respectively, and $I$ is the identity map, which of the following is true?
(A) $S T=I$
(B) $S T=-I$
(C) $T S=I$
(D) $S T=T S$
(E) $S T=-T S$
7. (Chapter 5 Prob 15) Let $T: \mathbb{R}^{5} \rightarrow \mathbb{R}^{3}$ be a linear transformation whose kernel is a threedimensional subspace of $\mathbb{R}^{5}$. The set $\left\{T(x): x \in \mathbb{R}^{5}\right\}$ is
(A) the trivial subspace
(B) a line through the origin
(C) a plane through the origin
(D) all of $\mathbb{R}^{3}$
(E) Cannot be determined from the information given
8. (Chapter 5 Prob 3) Let $A, B$ and $C$ be real $2 \times 2$ matrices, and let 0 denote the $2 \times 2$ zero matrix. Which of the following statements is/are true?
I. $A^{2}=0 \Rightarrow A=0$
II. $A B=A C \Rightarrow B=C$
III. $A$ is invertible and $A=A^{-1} \Rightarrow A=I$ or $A=-I$
(A) I only
(B) I and III only
(C) II and III only
(D) III only
(E) none of the above
9. (Week 4 Prob 11) If $V, W$ are 2-dimensional subspaces of $\mathbb{R}^{4}$, what are the possible dimensions of $V \cap W$ ?
(A) 0
(B) 0,1
(C) $0,1,2$
(D) 1,2
(E) 2
10. (Week 4 Prob 12) Suppose that $V$ is the vector space of real $2 \times 3$ matrices. If $T$ is a linear transformation from $V$ onto $\mathbb{R}^{4}$, what is the dimension of the null space of $T$ ?
(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

Answer: AEBD CECE CC


[^0]:    ${ }^{1}$ The problems with "Chapter $*$ " are taken from "Cracking the GRE Mathematics Test", 4th Edition.
    ${ }^{2}$ The problems with "Exam I" - "Exam VI" are taken from the REA book "The Best Test Preparation for the GRE Mathematics Test", 4th edition.

