

GRE Math Subject Prep Course: Linear Algebra

June 30, 2021

1. (Exam III Prob 3) Which of the following matrices is normal? ($i = \sqrt{-1}$)

(A) $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & i \\ -1 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}$

(D) $\begin{pmatrix} i & 1 \\ -1 & 0 \end{pmatrix}$ (E) $\begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$

2. (Exam III Prob 12) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by

$$T(x, y) = \begin{bmatrix} 2x - y \\ x + 3y \end{bmatrix}$$

Find the adjoint T^* of T .

(A) $\begin{bmatrix} 2x + y \\ -x + 3y \end{bmatrix}$ (B) $\begin{bmatrix} x + 2y \\ x - 3y \end{bmatrix}$ (C) $\begin{bmatrix} 2x + y \\ x - 3y \end{bmatrix}$

(D) $\begin{bmatrix} x/2 - y \\ -x + y/3 \end{bmatrix}$ (E) $\begin{bmatrix} 3x - y \\ x + 2y \end{bmatrix}$

3. (Exam IV Prob 64) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, and let I be an identity matrix. Which matrix polynomial is zero?

(A) $A^2 - 10A + I$ (B) $A^2 - 10A$ (C) $A^2 - 5A - 2I$

(D) $A^2 + 5A - 2I$ (E) $A^2 + 5A + 2I$

4. (Exam IV Prob 13) Given that 3 by 3 matrix A has only one eigenvalue, what is the dimension of the corresponding eigenspace?

(A) 1 (B) 2 (C) 3

(D) 1 or 2 (E) 1, 2 or 3

5. (Exam II Prob 32) If A is an $n \times n$ matrix with diagonal entries a and other entries b , then one eigenvalue of A is $a - b$. Find another eigenvalue of A .

(A) $b - a$ (B) $nb + a - b$ (C) $nb - a + b$
(D) 0 (E) none of these

6. (Exam I Prob 44) Let $M = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$. Then $M^6 = kM$ for $k =$

(A) 2^6 (B) 2^8 (C) 2^{10}
(D) 2^{12} (E) 2^{14}

7. (Exam IV Prob 58) If the determinants $|A| = 3$ and $|B| = 2$, find $|2(AB)^{-1}|$ for 4×4 matrices A and B .

(A) $1/3$ (B) $2/3$ (C) $4/3$
(D) $8/3$ (E) 12

8. (Exam VI Prob 35) Let A and B be $n \times n$ symmetric matrices. Which of the following is necessary and sufficient condition for AB to be symmetric?

(A) BA is skew-symmetric (B) A, B are nonsingular (C) $|AB| = |BA|$
(D) A and B commute (E) B is Hermitian

9. (Practice Prob 31) Of the number 2, 3 and 5, which are eigenvalues of the matrix $\begin{pmatrix} 3 & 5 & 3 \\ 1 & 7 & 3 \\ 1 & 2 & 8 \end{pmatrix}$

(A) NONE (B) 2 and 3 only (C) 2 and 5 only
(D) 3 and 5 only (E) 2, 3 and 5

10.

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix}$$

(Practice Prob 34) Which of the following statements about the real matrix shown above is FALSE?

- (A) A is invertible.
 - (B) If $x \in \mathbb{R}$ and $Ax = x$, then $x = 0$.
 - (C) The last row of A^2 is $(0\ 0\ 0\ 0\ 25)$
 - (D) A can be transformed into 5×5 identity matrix by a sequence of elementary row operations.
 - (E) $\det(A) = 120$
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11. (Practice Prob 37) Let V be a finite-dimensional real vector space and let P be a linear transformation of V such that $P^2 = P$. Which of the following must be true?

- I. P is invertible.
- II. P is diagonalizable.
- III. P is either the identity transformation or the zero transformation.

- (A) None
- (B) I only
- (C) II only
- (D) III only
- (E) II and III

Answer: DACE BCDD CBC