# GRE Math Subject Prep Course: Topology 

July 14, 2021

1. (Exam III Prob 27) ${ }^{1}$ Let $X=\{a, b, c\}$. Which of the following classes of subsets of $X$ does NOT form a topology on $X$ ?
(A) $\{X, \varnothing\}$
(B) $\{X, \varnothing,\{a\}\}$
(C) $\{X, \varnothing,\{a\},\{b\},\{a, b\}\}$
(D) $\{X, \varnothing,\{a, b\},\{a, c\},\{b, c\}\}$
(E) $P(X)$, the power set of $X$.
2. (Exam II Prob 21) How many topologies are possible on a set of 2 points?
(A) 5
(B) 4
(C) 3
(D) 2
(E) 1
3. (Exam II Prob 64) Let $S=\left\{x_{1}, x_{2}, \ldots, x_{n}, \ldots\right\}$ be a topological space where the open sets are $U_{n}=\left\{x_{1}, \ldots, x_{n}\right\}$ for $n=1,2, \ldots$ Let $E=\left\{x_{2}, x_{4}, \ldots, x_{2 k}\right\}$. Find the set of cluster points of $E$.
(A) $S-\left\{x_{1}, x_{2}\right\}$
(B) $\left\{x_{1}\right\}$
(C) $\left\{x_{2}\right\}$
(D) $E-\left\{x_{2}\right\}$
(E) $S-E$
4. (Exam III Prob 61) Which of the following is a neighborhood of 0 relative to the usual topology $\tau$ for the real numbers?
(A) $(0,1)$
(B) $[-1,1]$
(C) $[-1,0]$
(D) $[0,1]$
(E) $(-1,0)$

[^0]5. (Chapter 7 Prob 34$)^{2}$ Let $(X, T)$ be a topological space, and let $A$ be the subset $(0,1) \cup[4,6)$ in $\mathbb{R}$. Find the exterior of $A$.
(A) $(-\infty, 0) \cup(2,3) \cup(6, \infty)$
(B) $[0,1] \cup[4,6]$
(C) $(-\infty, 0) \cup(1,4) \cup(6, \infty)$
(D) $(-\infty, 0] \cup[1,4) \cup(6, \infty)$
(E) $(-\infty, 0) \cup(6, \infty)$
6. (Exam IV Prob 32) The set of all points in the plane satisfying $y=x \sin \left(\frac{1}{x}\right)$ together with the origin
(A) is compact but not connected
(B) is connected but not compact
(C) is compact and connected
(D) contains an open set
(E) does not contain all of its limit points
7. (Exam III Prob 65) If $\tau$ is the discrete topology on the real numbers $\mathbb{R}$, find the closure of $(a, b)$.
(A) $(a, b)$
(B) $(a, b]$
(C) $[a, b)$
(D) $[a, b]$
(E) $\mathbb{R}$
8. (Exam II Prob 55) Let $f$ be a mapping from a topological space $X$ onto itself. Which of the following is true for continuous $f$ ? (Hint: "onto" means $f$ is a surjection.)
(A) Every open set in $X$ is the image of an open set in $X$.
(B) $f^{-1}(B)$ is bounded for each bounded set $B$ in $X$.
(C) $f$ is one-to-one.
(D) Both (A) and (B)
(E) Both (A) and (C)

[^1]9. (Exam VI Prob 60) Which of the following sets in $\mathbb{R}^{2}$ are compact?
(A) $\{x, y: x \geq 0, y \geq 0\}$
(B) $\{x, y: 0 \leq x \leq 1,0 \leq y \leq 1\}$
(C) $\{x, y:|x-y| \leq 2\}$
(D) $\left\{x, y: x^{2}+y^{2}<2\right\} \cap\left\{x, y: x^{2}+y^{2}>1\right\}$
(E) $\{x, y:|x+y| \leq 1\}$
10. (Practice Book Prob 56$)^{3}$ For every set $S$ and every metric $d$ on $S$, which of the following is a metric on $S$ ?
(A) $4+d$
(B) $e^{d}-1$
(C) $d-|d|$
(D) $d^{2}$
(E) $\sqrt{d}$
11. (Exam III Prob 18) Let $R[0,1]$ denote the set of Riemann integrable functions defined on $[0,1]$. Which of the following is NOT satisfied by the function $d$ defined on $R[0,1]$ by
$$
d(f, g)=\int_{0}^{1}|f(x)-g(x)| d x ?
$$
(A) $d(f, f)=0$
(B) $d(f, g) \geq 0$
(C) $d(f, g)>0$ if $f \neq g$
(D) $d(f, g)=d(g, f)$
(E) $d(f, g) \leq d(f, h)+d(h, g)$
12. (Exam IV Prob 52) Let $C_{n}$ be a sequence of closed, bounded, nonempty intervals in the real line with the usual topology. The intervals are also nested in the sense that $C_{n+1} \subseteq C_{n}$.
Which of the following is true of the intersection $S=\bigcap_{k=1}^{\infty} C_{k}$ ?
(A) $S$ may be open or closed.
(B) $S$ may be empty.

[^2](C) $S$ must be nonempty and closed.
(D) $S$ must contain an interval.
(E) $S$ must not contain an interval.
13. (Week 6 Prob 14) Let $\tau$ be the topology on $\mathbb{R}$ generated by sets of the form $\{[a, b): a, b \in$ $\mathbb{R}, a<b\}$. Which of the following are true in the topological space $(\mathbb{R}, \tau)$ ?
I. $[0,1]$ is compact.
II. $[0,1]$ is Hausdorff.
III. $[0,1]$ is connected.
(A) I and II only
(B) II and III only
(C) I only
(D) II only
(E) None of above
14. (Week 6 Prob 15) Let $S \subset[0,1] \times[0,1]$ consist of all points $(x, y) \in[0,1] \times[0,1]$ such that $x$ or $y$ or both is irrational. Which of the following is true (with respect to the standard topology on $\left.\mathbb{R}^{2}\right)$ ?
I. $S$ is open.
II. $S$ is closed.
III. $S$ is connected.
(A) I and II only
(B) II and III only
(C) I only
(D) II only
(E) III only

Answer: DBAB CBAA BECC DE


[^0]:    ${ }^{1}$ The problems with "Exam I" - "Exam VI" are taken from the REA book "The Best Test Preparation for the GRE Mathematics Test", 4th edition.

[^1]:    ${ }^{2}$ The problems with "Chapter *" are taken from "Cracking the GRE Mathematics Test", 4th Edition.

[^2]:    ${ }^{3}$ The problems with "Practice Book" are taken from the mathematics test practice book by ETS, which can be found at http://www.ets.org/Media/Tests/GRE/pdf/Math.pdf

