## GRE Math Subject Prep Course: Topology

## July 14, 2021

- 1. (Exam III Prob 27)<sup>1</sup> Let  $X = \{a, b, c\}$ . Which of the following classes of subsets of X does NOT form a topology on X?
  - (A)  $\{X, \emptyset\}$
  - (B)  $\{X, \emptyset, \{a\}\}$
  - (C)  $\{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$
  - (D)  $\{X, \emptyset, \{a, b\}, \{a, c\}, \{b, c\}\}$
  - (E) P(X), the power set of X.

2. (Exam II Prob 21) How many topologies are possible on a set of 2 points?

(A) 5	(B) 4	(C) 3
(D) 2	(E) 1	

3. (Exam II Prob 64) Let  $S = \{x_1, x_2, \ldots, x_n, \ldots\}$  be a topological space where the open sets are  $U_n = \{x_1, \ldots, x_n\}$  for  $n = 1, 2, \ldots$  Let  $E = \{x_2, x_4, \ldots, x_{2k}\}$ . Find the set of cluster points of E.

(A) $S - \{x_1, x_2\}$	(B) $\{x_1\}$	(C) $\{x_2\}$
(D) $E - \{x_2\}$	(E) $S-E$	

4. (Exam III Prob 61) Which of the following is a neighborhood of 0 relative to the usual topology  $\tau$  for the real numbers?

(A) (0,1) (B) [-1,1] (C) [-1,	(A) (0,1)	(B) $[-1,1]$	(C) $[-1, 0]$
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(D) [0,1] (E) (-1,0)

 $<sup>^1{\</sup>rm The}$  problems with "Exam I" – "Exam VI" are taken from the REA book "The Best Test Preparation for the GRE Mathematics Test", 4th edition.

- 5. (Chapter 7 Prob 34)<sup>2</sup> Let (X, T) be a topological space, and let A be the subset  $(0, 1) \cup [4, 6)$  in  $\mathbb{R}$ . Find the exterior of A.
  - (A)  $(-\infty,0) \cup (2,3) \cup (6,\infty)$
  - (B)  $[0,1] \cup [4,6]$
  - (C)  $(-\infty, 0) \cup (1, 4) \cup (6, \infty)$
  - (D)  $(-\infty,0] \cup [1,4) \cup (6,\infty)$
  - (E)  $(-\infty, 0) \cup (6, \infty)$
- 6. (Exam IV Prob 32) The set of all points in the plane satisfying  $y = x \sin\left(\frac{1}{x}\right)$  together with the origin
  - (A) is compact but not connected
  - (B) is connected but not compact
  - (C) is compact and connected
  - (D) contains an open set
  - (E) does not contain all of its limit points
- 7. (Exam III Prob 65) If  $\tau$  is the discrete topology on the real numbers  $\mathbb{R}$ , find the closure of (a, b).

(A) $(a,b)$	(B) $(a,b]$	(C) $[a,b)$
(D) $[a,b]$	(E) $\mathbb{R}$	

- 8. (Exam II Prob 55) Let f be a mapping from a topological space X onto itself. Which of the following is true for continuous f? (Hint: "onto" means f is a surjection.)
  - (A) Every open set in X is the image of an open set in X.
  - (B)  $f^{-1}(B)$  is bounded for each bounded set B in X.
  - (C) f is one-to-one.
  - (D) Both (A) and (B)
  - (E) Both (A) and (C)

<sup>&</sup>lt;sup>2</sup>The problems with "Chapter \*" are taken from "Cracking the GRE Mathematics Test", 4th Edition.

- 9. (Exam VI Prob 60) Which of the following sets in  $\mathbb{R}^2$  are compact?
  - (A)  $\{x, y : x \ge 0, y \ge 0\}$
  - (B)  $\{x, y: 0 \le x \le 1, 0 \le y \le 1\}$
  - (C)  $\{x, y : |x y| \le 2\}$
  - (D)  $\{x, y: x^2 + y^2 < 2\} \cap \{x, y: x^2 + y^2 > 1\}$
  - (E)  $\{x, y : |x+y| \le 1\}$
- 10. (Practice Book Prob 56)<sup>3</sup> For every set S and every metric d on S, which of the following is a metric on S?

(A) $4+d$	(B) $e^d - 1$	(C) $d -  d $
(D) $d^2$	(E) $\sqrt{d}$	

11. (Exam III Prob 18) Let R[0,1] denote the set of Riemann integrable functions defined on [0,1]. Which of the following is NOT satisfied by the function d defined on R[0,1] by

$$d(f,g) = \int_0^1 |f(x) - g(x)| dx ?$$

- (A) d(f, f) = 0
- (B)  $d(f,g) \ge 0$
- (C) d(f,g) > 0 if  $f \neq g$
- (D) d(f,g) = d(g,f)
- (E)  $d(f,g) \le d(f,h) + d(h,g)$
- 12. (Exam IV Prob 52) Let  $C_n$  be a sequence of closed, bounded, nonempty intervals in the real line with the usual topology. The intervals are also nested in the sense that  $C_{n+1} \subseteq C_n$ .

Which of the following is true of the intersection  $S = \bigcap_{k=1}^{\infty} C_k$ ?

- (A) S may be open or closed.
- (B) S may be empty.

 $<sup>^{3}</sup>$ The problems with "Practice Book" are taken from the mathematics test practice book by ETS, which can be found at http://www.ets.org/Media/Tests/GRE/pdf/Math.pdf

- (C) S must be nonempty and closed.
- (D) S must contain an interval.
- (E) S must not contain an interval.
- 13. (Week 6 Prob 14) Let  $\tau$  be the topology on  $\mathbb{R}$  generated by sets of the form  $\{[a, b) : a, b \in \mathbb{R}, a < b\}$ . Which of the following are true in the topological space  $(\mathbb{R}, \tau)$ ?
  - I. [0,1] is compact.
  - II. [0,1] is Hausdorff.
  - III. [0,1] is connected.

(A) I and II only	(B) II and III only	(C) I only
(D) II only	(E) None of above	

- 14. (Week 6 Prob 15) Let  $S \subset [0,1] \times [0,1]$  consist of all points  $(x, y) \in [0,1] \times [0,1]$  such that x or y or both is irrational. Which of the following is true (with respect to the standard topology on  $\mathbb{R}^2$ )?
  - I. S is open.
  - II. S is closed.
  - III. S is connected.
    - (A) I and II only (B) II and III only (C) I only
    - (D) II only (E) III only

## Answer: DBAB CBAA BECC DE