On Contact Invariants in Bordered Floer Homology

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Contact structures and Heegaard Floer homology

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ξ overtwisted $\Rightarrow$ invariants vanish

Bordered Sutured Contact Invariants

Theorem 1 (Min–V.)

Let $(Y, \Gamma, \mathcal{F})$ be a bordered sutured manifold and ξ a contact structure on the sutured manifold $(Y, \Gamma \cup \Gamma_I)$ where Γ_I is an elementary dividing set for $\mathcal{F}$. There exist contact invariants: $c_A(\xi) \in \widehat{BSA}(-Y)$ and $c_D(\xi) \in \widehat{BSD}(\mathcal{TW}^+ \cup -Y)$

(More generally, bimodule invariants: $c_{AA}(\xi) \in \widehat{BSAA}(-Y)$, etc.)

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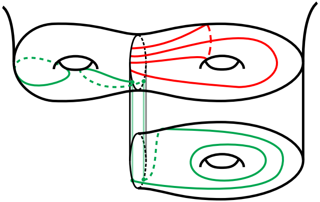

Properties

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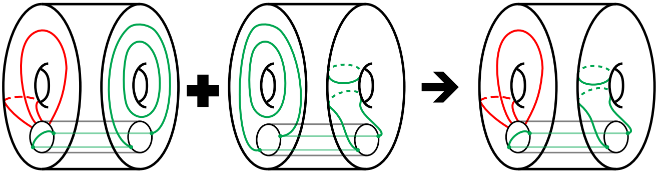

Computations

- c_A and c_D are derived from EH

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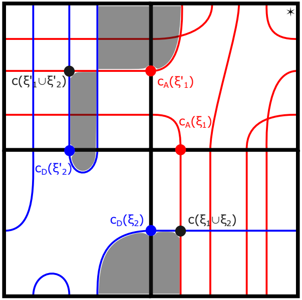
- For torus boundary, amenable to immersed curve technique of [HRW24]



Applications

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- Invariants of Legendrian satellite knots?

Thank you!

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