Contact structures on hyperbolic 3-manifolds

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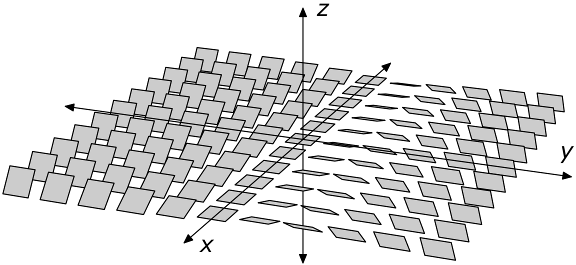
Joint work with James Conway

Contact structures

- A contact structure on a 3-manifold M is a plane field $\xi = \ker \alpha$ where $\alpha \in \Omega^1(M)$, $\alpha \wedge d\alpha > 0$.



Tight & overtwisted contact structures

- An overtwisted disk is an embedded disk tangent to the contact planes along the boundary.
- A contact structure is called overtwisted if it contains an overtwisted disk.
- A contact structure is called tight if it does not contain an overtwisted disk.

Theorem (Eliashberg 1989)

There is a one to one correspondence between overtwisted contact structures (up to isotopy) and plane fields (up to homotopy).

Tight contact structures

Goal
Classify tight contact structures up to isotopy.

Prime manifolds

- Seifert fibration: Many results
- Toroidal: Many results
- Hyperbolic: No result

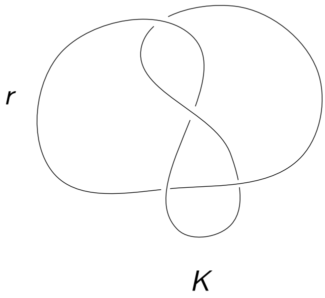
Hyperbolic manifolds

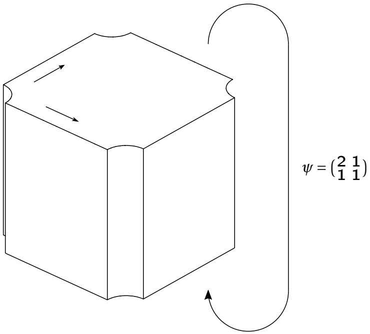
Why is it hard?

- Need a 'good' decomposition.
- Analyze contact structures in each piece.
- Most decompositions for hyperbolic manifolds are not simple enough.

Figure-8 knot

- Surgeries on the figure-8 knot yield hyperbolic manifolds.
- $S^3_r(K) \setminus N(K^*)$ is a punctured torus bundle over S^1 with a pseudo-Anosov monodromy.





Results I

Theorem (M – Conway 2018)

There are exactly two Stein fillable and universally tight contact structures on $S^3_{1/n}(K)$ for $n < -1$.

Results II

Theorem (M – Conway 2018)

Let r be a rational number. Then $S_r(K)$ supports

$$\begin{cases} 2\Phi(r), & r \in [1,4) \cup [5,\infty) \\ \Phi(r) + \Psi(r), & r \in (-\infty,-4) \cup [-3,0) \end{cases}$$

tight contact structures, where

$$-\frac{1}{r} = [r_0, \ldots, r_n] = r_0 - \cfrac{1}{r_1 - \cfrac{1}{\ddots - \cfrac{1}{r_n}}},$$

$$\Phi(r) = \left| r_0(r_1+1)\cdots(r_n+1) \right|.$$

$$\Psi(r) = \begin{cases} 0, & r \geq -3 \\ \Phi(-\frac{1}{r+3}), & r < -3. \end{cases}$$

Thank you!