Contact structures on hyperbolic 3-manifolds

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Joint work with James Conway
A contact structure on a 3-manifold $M$ is a plane field $\xi = \ker \alpha$ where $\alpha \in \Omega^1(M)$, $\alpha \wedge d\alpha > 0$. 

Diagram:

- A grid of arrows showing the direction of the plane field $\xi$. 
- The arrows are distributed in such a way that they are orthogonal to the $z$-axis and aligned with the $x$- and $y$-axes, indicating the contact structure.

This diagram illustrates the concept of a contact structure, where the plane field is oriented in a specific way within the 3-dimensional space.
An overtwisted disk is an embedded disk tangent to the contact planes along the boundary.

A contact structure is called overtwisted if it contains an overtwisted disk.

A contact structure is called tight if it does not contain an overtwisted disk.

**Theorem (Eliashberg 1989)**

*There is a one to one correspondence between overtwisted contact structures (up to isotopy) and plane fields (up to homotopy).*
Tight contact structures

Goal
Classify tight contact structures up to isotopy.

Prime manifolds

- Seifert fibration: Many results
- Toroidal: Many results
- Hyperbolic: No result
Why is it hard?

- Need a 'good' decomposition.
- Analyze contact structures in each piece.
- Most decompositions for hyperbolic manifolds are not simple enough.
Figure-8 knot

- Surgeries on the figure-8 knot yield hyperbolic manifolds.
- \( S^3_r(K) \setminus N(K^*) \) is a punctured torus bundle over \( S^1 \) with a pseudo-Anosov monodromy.

\[ \psi = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \]
Theorem (M – Conway 2018)

There are exactly two Stein fillable and universally tight contact structures on $S_{1/n}^3(K)$ for $n < -1$. 
Theorem (M – Conway 2018)

Let \( r \) be a rational number. Then \( S_r(K) \) supports

\[
\begin{cases} 
2\Phi(r), & r \in [1, 4) \cup [5, \infty) \\
\Phi(r) + \Psi(r), & r \in (-\infty, -4) \cup [-3, 0)
\end{cases}
\]

tight contact structures, where

\[
-\frac{1}{r} = \left[ r_0, \ldots, r_n \right] = r_0 - \frac{1}{r_1} - \frac{1}{r_2} - \cdots - \frac{1}{r_n},
\]

\[
\Phi(r) = \left| r_0 (r_1 + 1) \cdots (r_n + 1) \right|.
\]

\[
\Psi(r) = \begin{cases} 
0, & r \geq -3 \\
\Phi\left(-\frac{1}{r+3}\right), & r < -3.
\end{cases}
\]
Thank you!