Cabling Legendrian knots

Hyunki Min
with Apratim Chakraborty and John Etnyre

Tech Topology Conference
December 6, 2019

Legendrian knots in $(\mathbb{R}^3, \xi_{std})$

- A standard contact structure on $\mathbb{R}^3$ is a plane field

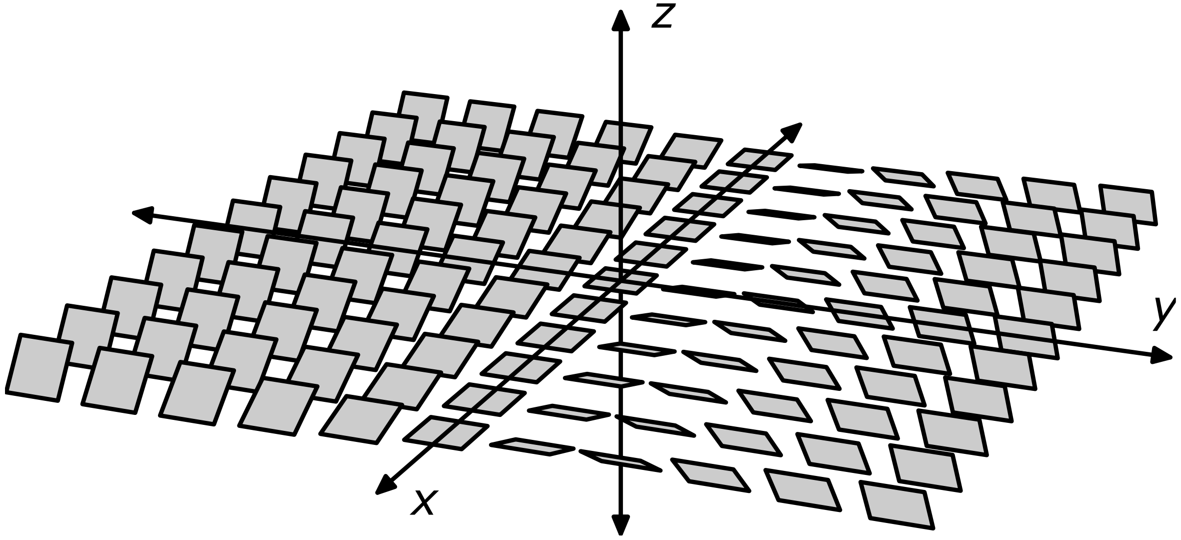
$$\xi = \ker dz - ydx$$

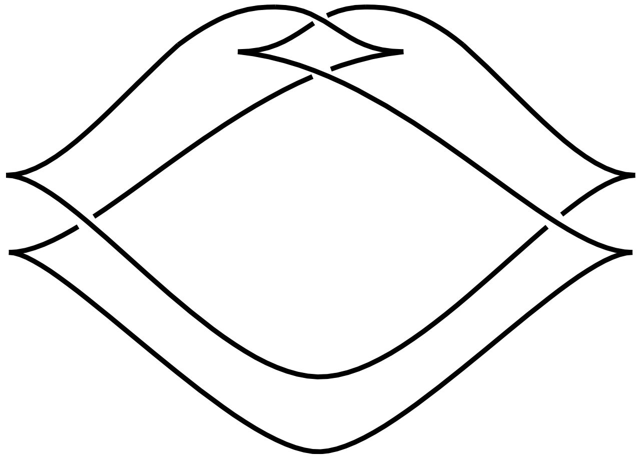
Legendrian knots in $(\mathbb{R}^3, \xi_{std})$

- A standard contact structure on $\mathbb{R}^3$ is a plane field
$$\xi = \ker dz - ydx$$
- A Legendrian knot is a knot tangent to the contact structure.

Legendrian knots in $(\mathbb{R}^3, \xi_{std})$

- A standard contact structure on $\mathbb{R}^3$ is a plane field
$$\xi = \ker dz - ydx$$
- A Legendrian knot is a knot tangent to the contact structure.





Classical invariants

- Thurston-Bennequin invariant
  Contact framing - Seifert framing

Classical invariants

- Thurston-Bennequin invariant
  Contact framing - Seifert framing

- Rotation number
  A winding number of a tangent field with respect to a trivialization of $\xi|_\Sigma$

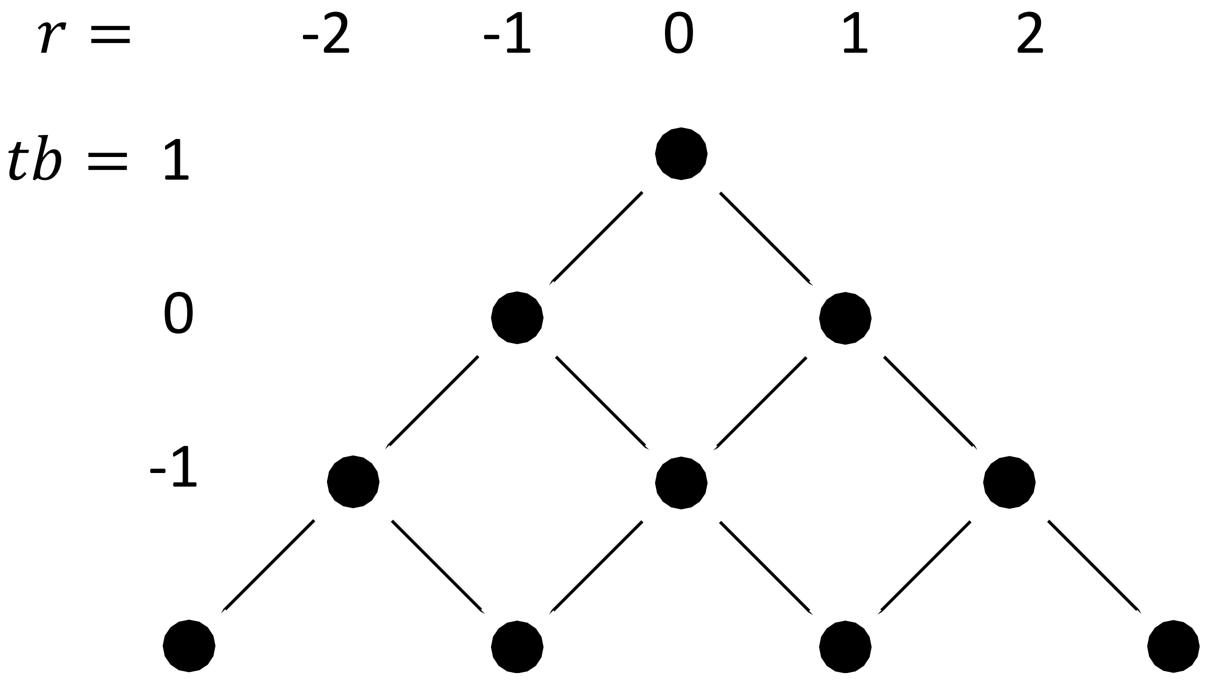
Classical invariants

- Thurston-Bennequin invariant
  Contact framing - Seifert framing

- Rotation number
  A winding number of a tangent field with respect to a trivialization of $\xi|_\Sigma$

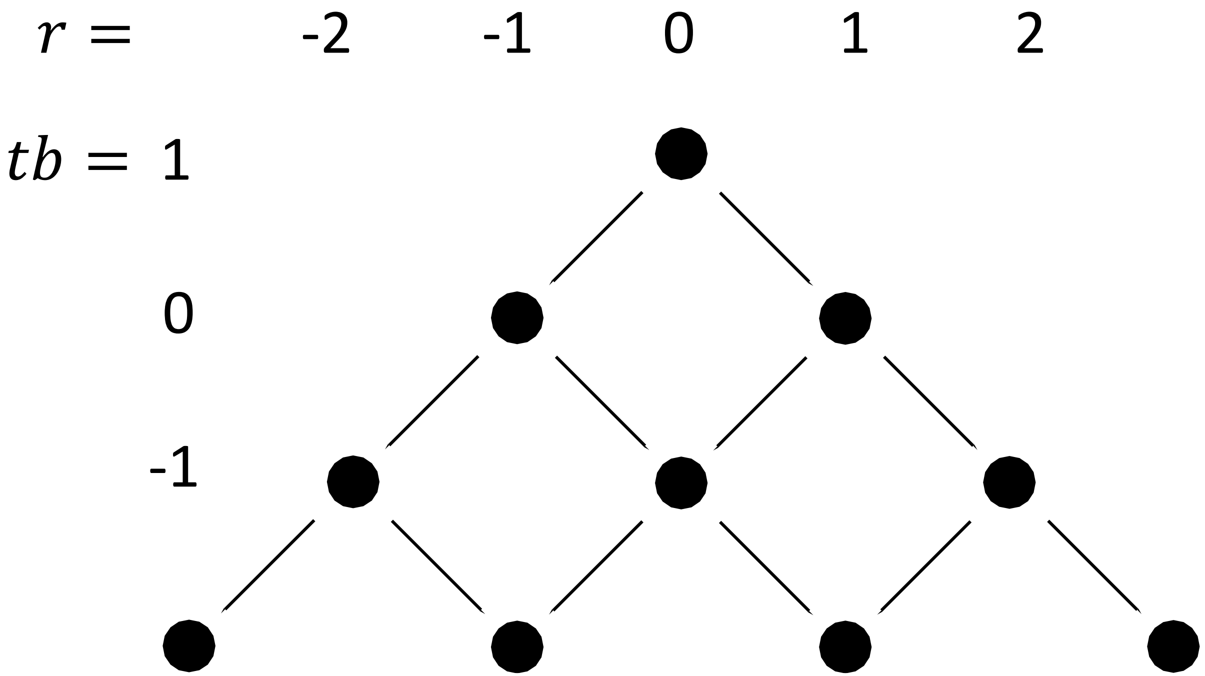
- A knot is called Legendrian simple if its Legendrian isotopy classes are determined by Thurston-Bennequin and rotation numbers.

Mountain range

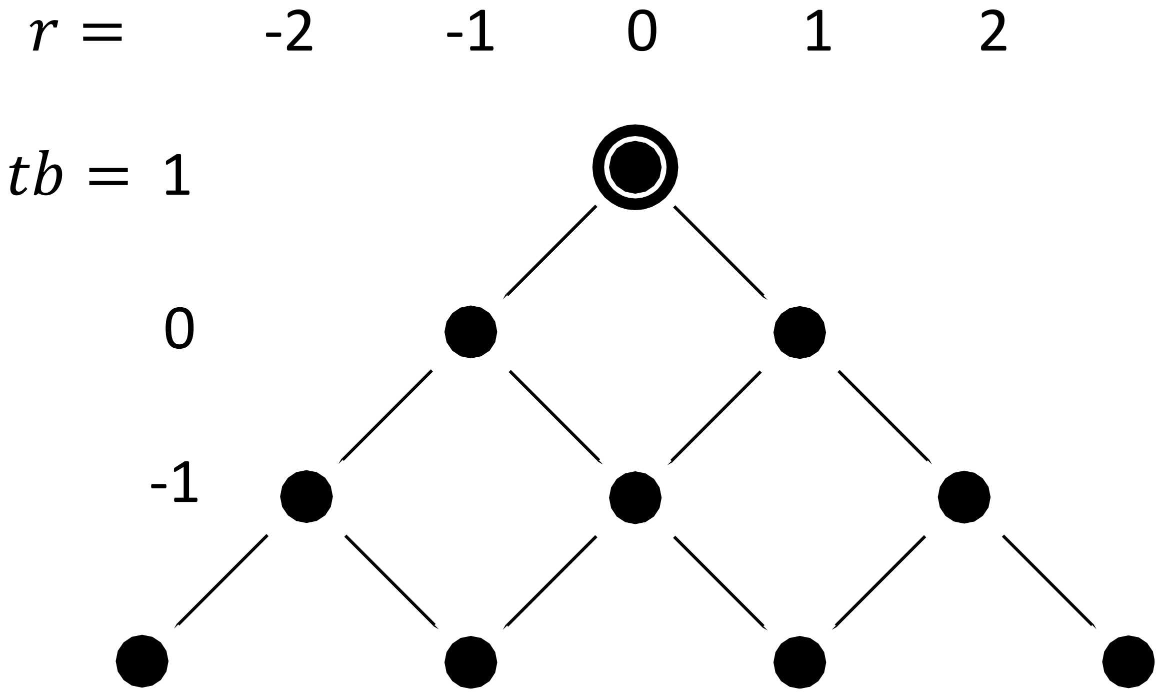


Right-handed trefoil

Mountain range



Right-handed trefoil



$m5_2$

Legendrian cables

- Question: If the classification of Legendrian knots of a given knot type is known, is it possible to classify Legendrian knots of its cables?

Legendrian cables

- Question: If the classification of Legendrian knots of a given knot type is known, is it possible to classify Legendrian knots of its cables?

- (Etnyre-Honda) If K is Legendrian simple and uniformly thick, then $K_{p,q}$ is also Legendrian simple for any (p, q)

Legendrian cables

- Question: If the classification of Legendrian knots of a given knot type is known, is it possible to classify Legendrian knots of its cables?

- (Etnyre-Honda) If K is Legendrian simple and uniformly thick, then $K_{p,q}$ is also Legendrian simple for any (p, q)
- (Tosun) If K is Legendrian simple, then $K_{p,q}$ is also Legendrian simple for $q/p > \overline{tb}(K) + 1$

Legendrian cables

- Question: If the classification of Legendrian knots of a given knot type is known, is it possible to classify Legendrian knots of its cables?

- (Etnyre-Honda) If K is Legendrian simple and uniformly thick, then $K_{p,q}$ is also Legendrian simple for any (p, q)
- (Tosun) If K is Legendrian simple, then $K_{p,q}$ is also Legendrian simple for $q/p > \overline{tb}(K) + 1$
- (Chakraborty) $\hat{\theta}(T^1) = \hat{\theta}(T^2)$ if and only if $\hat{\theta}\left(T^1_{p,q}\right) = \hat{\theta}\left(T^2_{p,q}\right)$

Main result

Theorem (Chakraborty-Etnyre-M)

- For $q/p > \overline{tb}(K) + 1$, $K_{p,q}$ is Legendrian simple if and only if K is Legendrian simple.

Main result

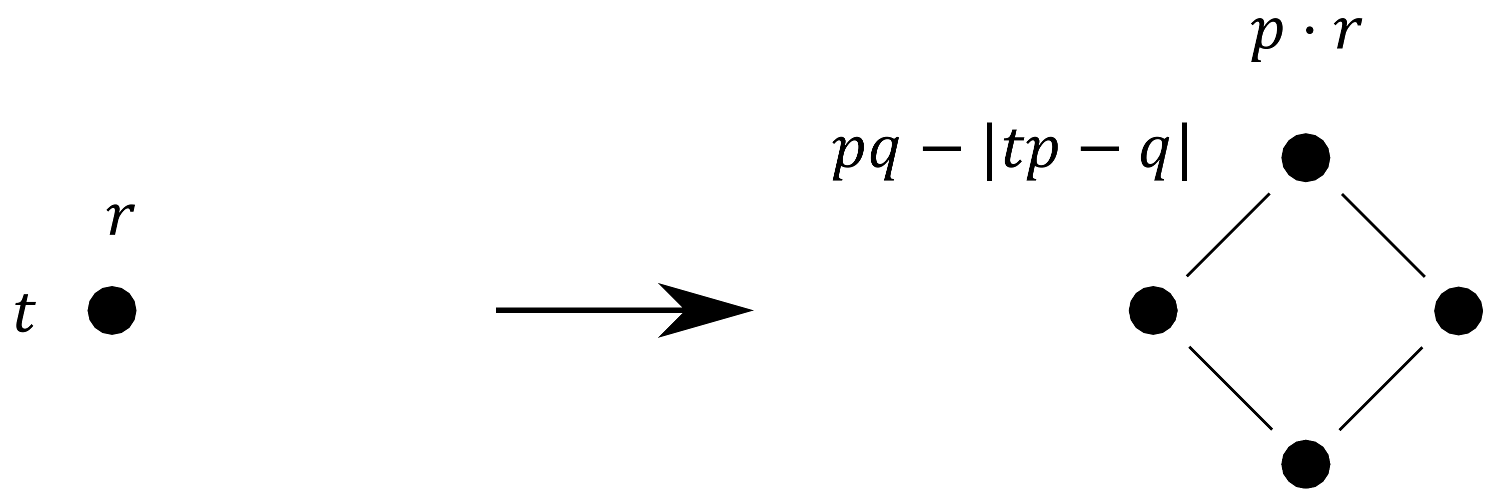
Theorem (Chakraborty-Etnyre-M)

- For $q/p > \overline{tb}(K) + 1$, $K_{p,q}$ is Legendrian simple if and only if K is Legendrian simple.
- The mountain range of $K_{p,q}$ is a (p, q)-diamond of the mountain range of K.

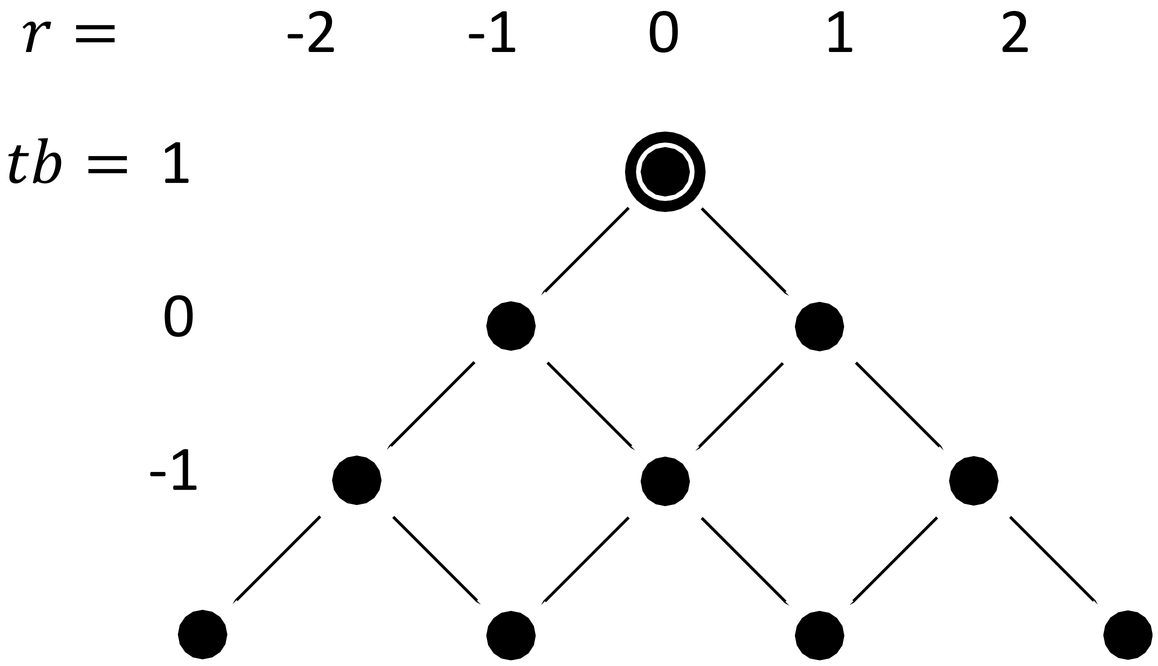
Main result

Theorem (Chakraborty-Etnyre-M)

- For $q/p > \overline{tb}(K) + 1$, $K_{p,q}$ is Legendrian simple if and only if K is Legendrian simple.
- The mountain range of $K_{p,q}$ is a (p,q)-diamond of the mountain range of K.

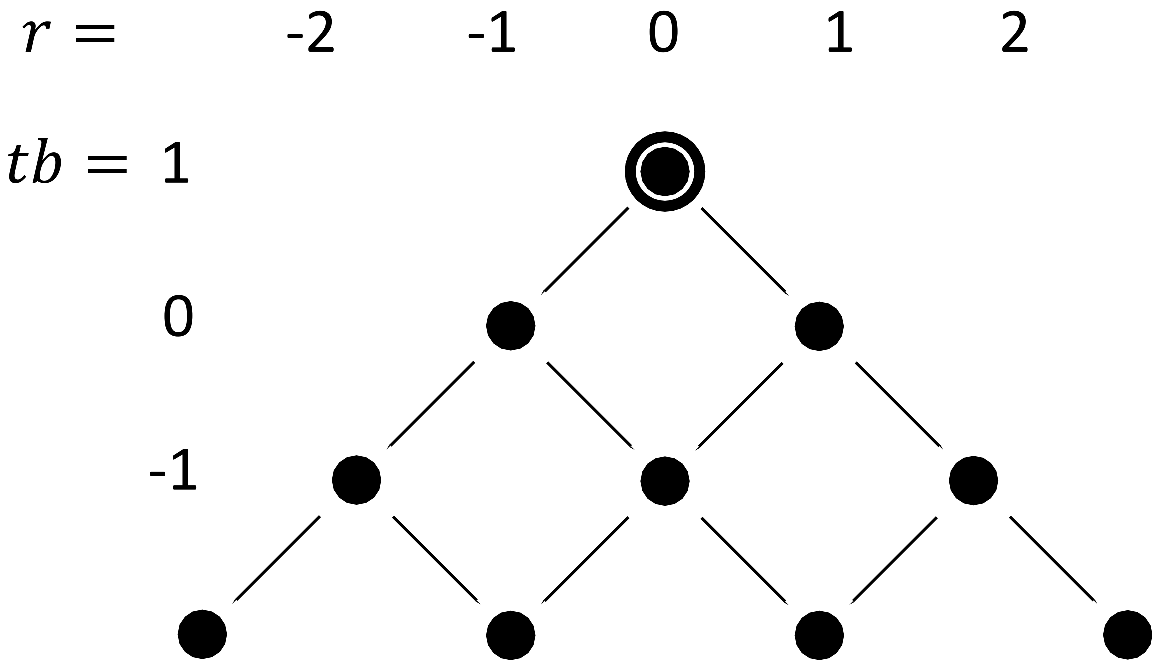


Examples

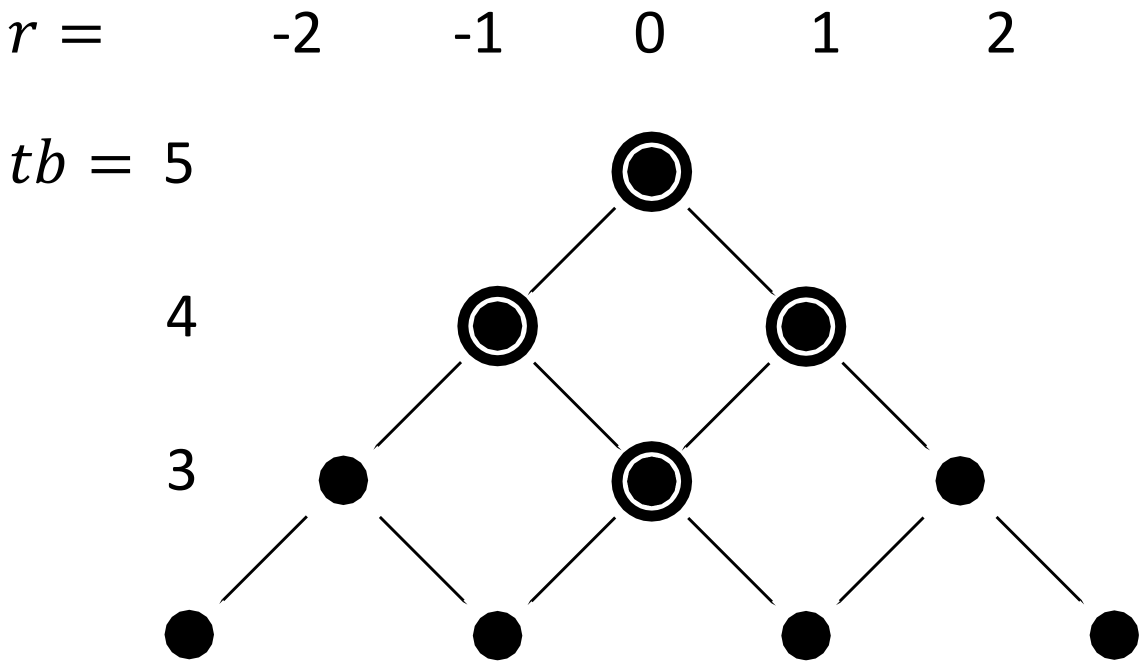


$m5_2$

Examples



$m5_2$



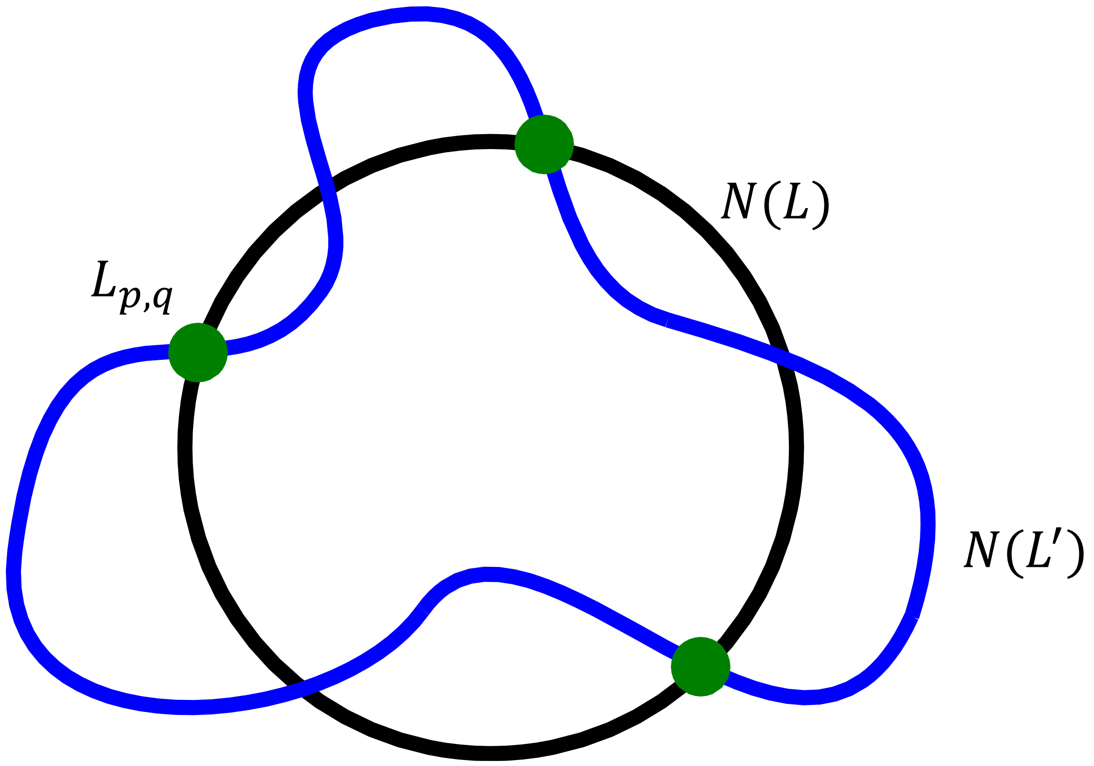
$(2,3)$-cable of $m5_2$

Idea of Proof

- Put a cable $L_{p,q}$ on a standard neighborhood of L.
- Assume we have a common $L_{p,q}$ on neighborhoods of L and L'

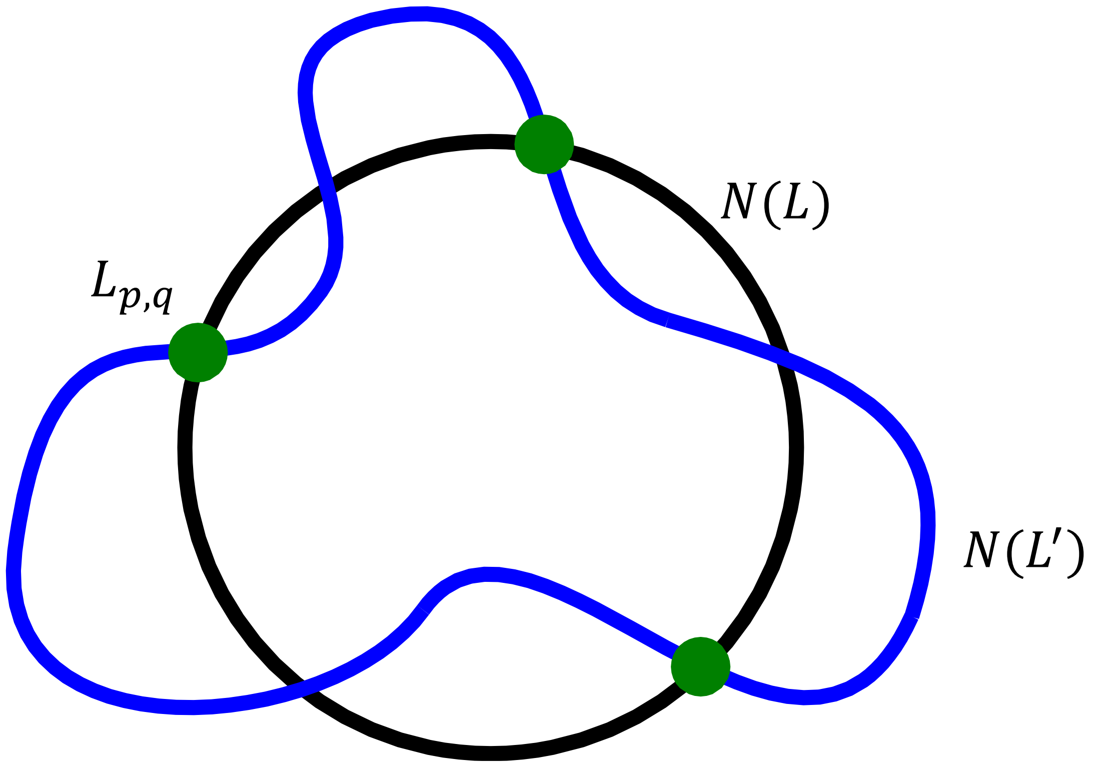
Idea of Proof

- Put a cable $L_{p,q}$ on a standard neighborhood of L.
- Assume we have a common $L_{p,q}$ on neighborhoods of L and L'



Idea of Proof

- There is a smooth isotopy from $N(L)$ to $N(L')$ fixing $L_{p,q}$
- Keep track of the contact structure on $N(L)$ during the isotopy



Thank you!